Week 2 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
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   or reach out on Discourse: `@23f3004032`

Quiz 1 May 22

1. For what value of k is the function continuous?

$$f(x) = \begin{cases} 3x^2 - kx + 5, & 0 \leq x < 2 \\ 1 - 3x, & 2 \leq x \leq 3 \end{cases}$$

[Ans: 11]

2. Consider a function

$$f(x) = \begin{cases} \dfrac{x - |x|}{x}, & \text{if } x \neq 0 \\ 2, & \text{if } x = 0 \end{cases}$$

Is f continuous?

(Provide 1 as answer for 'Yes' and 0 for 'No'.)

[Ans:0]

Quiz 2 May 22

ET May 22

1. Consider the function $f(x, y) = \sqrt{41 - 4x^2 - y^2}$. Compute the equation of the function's linear approximation at point (2,3). If the equation of the linear approximation is $L = ax + by + c$, then provide the value of a. *NOTE: Enter your answer to the nearest integer.* [Ans: -2]

2. Consider the function $f(x, y) = \sqrt{41 - 4x^2 - y^2}$. Compute the equation of the function's linear approximation at point (2,3). If the equation of the linear approximation is $L = ax + by + c$, then provide the value of b. *NOTE: Enter your answer upto two decimal.* [Ans: -0.8 to -0.7]

3. Consider the function $f(x, y) = \sqrt{41 - 4x^2 - y^2}$. Compute the equation of the function's linear approximation at point (2,3). If the equation of the linear approximation is $L = ax + by + c$, then provide the value of c. *NOTE: Enter your answer upto two decimal.* [Ans: 10.0 to 10.5]

Quiz 1 Sep 22

1. For what value of k is the function continuous?

$$f(x) = \begin{cases} \frac{\tan 3x}{\sin 2x}, & x \neq 0 \\ 2k, & x = 0 \end{cases}$$

[Ans: 0.75]

2. Is the following function continuous?

$$f(x) = |x| \quad \text{at } x = 0$$

(Provide 1 as answer for 'Yes' and 0 for 'No'.)

[Ans: 1]

Quiz 2 Sep 22

ET Sep 22

1. Does the limit for the following function exist at r tends to zero? $f(x) = \sin \frac{1}{x}, x \neq 0$ (Provide 1 as answer for 'Yes' and 0 for 'No'.) [Ans: 0]

2. Which of the following statements are true?

   (a) If a function f is continuous at a, then $|f|$ is also continuous at a

   (b) If f is a continuous function at $x = a$ and g is a continuous function at $f(a)$ then gof is not a continuous function at a

   (c) If functions f and g be continuous at a, then $f + g$ is continuous at a

   (d) If functions f and g be continuous at a, then fg is continuous at a

   [Ans: 1, 3, 4]

Quiz 1 Jan 23

1. Which of the following statements is/are true about function

$$f(x) = \begin{cases} \frac{|x|}{x^2}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

   (a) The function is differentiable for all values of x except when $x = 0$.

   (b) The function is differentiable for all values of x.

   (c) The function is continuous at $x = 0$.

   (d) The function is not continuous at $x = 0$.

[Ans: a, d]

Quiz 2 Jan 23

ET Jan 23

1. If $f(x, y, z) = e^{(x+y+z)^2}$, Use the linear approximation of f around (0,0,1). Evaluate linear approximation at (1, 1, 1).

   (a) 3e

   (b) 2e

   (c) 4e

   (d) 5e

[Ans: 4e]

Quiz 1 May 23

1. Find the linear approximation of the function $f(x, y) = ye^{xy}$ in the neighbourhood of $(0, 1)$.

   (a) $x + y$

   (b) $x + 2y$

   (c) $2x + y$

   (d) $2x + 3y$

   [Ans: a]

2. Let

$$f(x) = \begin{cases} ax - 3, & x < 2 \\ 4, & x = 2 \\ 2x, & x > 2 \end{cases}$$

   be continuous. Find a.

   [Ans: 3.5]

Quiz 2 May 23

ET May 23

1. Find the linearization of $f(x, y) = e^{2y-x}$ at the point (1, 2).

   (a) $-e^3x + e^3y - e^3$

   (b) $-2e^3 + e^3x + 2e^3y$

   (c) $-e^3x + 2e^3y + e^3$

   (d) $-e^3x + 2e^3y$

[Ans: 4]

Quiz 1 Sep 23

1. The curve $y = ax^2 + bx + c$ passes through $(2, -1)$ and $(0, 0)$ and is tangent to $y = -x$ at origin. Find $|ab|$.

   [Ans: 0.25]

2. Find second order approximation of

   $$f(x, y) = e^{2y-x}$$

   near $(0, 0)$.

   (a) $1 - x + 2y - 2xy + y^2 + \frac{x^2}{2}$

   (b) $1 - 2xy + 2y^2 + \frac{x^2}{2}$

   (c) $2y^2 + \frac{x^2}{2}$

   (d) $1 - x + 2y - 2xy + 2y^2 + \frac{x^2}{2}$

   [Ans: d]

3. Let

   $$f(x) = \begin{cases} e^{-1/x^2}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

   Select which among the following are true?

   (a) f is discontinuous at $x = 0$

   (b) f is differentiable

   (c) f is not differentiable at $x = 0$

   (d) $f(0) = 0$

   [Ans: b, d]

Quiz 2 Sep 23

ET Sep 23

1. What is the linear approximation of $f(x, y) = e^{xy}$ around (1, 1)?

   (a) $e(x + y - 1)$

   (b) $x + y - 1$

   (c) $e(x + y + 1)$

   (d) $ex + ey$

[Ans: 1]

Quiz 1 Jan 24

1. Which of the following is the best approximation of $\sqrt[4]{16.1}$? (Use linear approximation around 16.)

   Options:

   (a) 2.1

   (b) 2.003125

   (c) 2.031

   (d) 1.9968

   [Ans: b]

2. Consider a function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = |x(x-2)|$. Which of the following options is/are true?

   (a) $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

   (b) f is differentiable at $x = 0$

   (c) $f'(3) = 4$

   (d) f is continuous at $x = 0$

   [Ans: a, c, d]

Quiz 2 Jan 24

ET Jan 24

1. Consider a function

$$f(x) = \begin{cases} 2x^2 & \text{if } x < 2 \\ (x-2)^3 & \text{if } 2 \le x < 4 \\ 4 & \text{if } 4 \le x \end{cases}$$

Based on the above data, answer the given subquestions.

Find the number of discontinuous points. [Ans: 2]

2. Find the number of points where the function is not differentiable. [Ans: 2]

Quiz 1 May 24

1.
$$f(x) = \begin{cases} ax^2 + bx + 1, & x < 0 \\ cx + d, & x \geq 0 \end{cases}$$
is Differentiable at $x = 0$.if a, b ,c ,d are constants which of the option must be true ?

(a) $a = c, b = 1$

(b) $b = c, d = 1$

(c) $a = c, d = 1$

(d) $b = d = 1$

[Ans: b]

2. Which of the following functions cannot be approximated linearly around $x = 0$?

(a) $f(x) = \sin(x)$

(b) $f(x) = \cos(x)$

(c) $f(x) = \ln(x)$

(d) $f(x) = x^2$

[Ans: c]

Quiz 2 May 24

ET May 24

Quiz 1 Sep 24

1. Determine which of the following limits yields a finite value.

   Options:

   (a) $\lim\limits_{x\to 1} \dfrac{x^{1/3}-1}{x^{1/2}-1}$

   (b) $\lim\limits_{x\to 1} \dfrac{1-x^{1/2}}{1-x}$

   (c) $\lim\limits_{x\to \infty} \dfrac{x^2-7x}{x+1}$

   (d) $\lim\limits_{x\to a} \dfrac{x^2-a^2}{x^4-a^4}$, where $a>0$

   (e) None of these

   [Ans: a,b,d]

2. For what value of b is

$$f(x)=\begin{cases}\dfrac{x-b}{b+1}, & x<0\\ x^2+b, & x\geq 0\end{cases}$$

   continuous at every x?

   (a) 3

   (b) 2

   (c) -2

   (d) 1

   [Ans: c]

3. Consider the function $f(x,y)=e^x\cos(y)$. Let $L(x,y)$ be the linear approximation of $f(x,y)$ at $(0,0)$, then find $L(0.5,0)$. Enter the answer correct to one decimal place.

   [Ans: 1.5]

4. Let $f:\mathbb{R}\to\mathbb{R}$ be the function, $f(x)=1+e^{-x}$. What is the value of the derivative of f at x where $f(x)=1.4$? Enter the answer correct to one decimal place.

   [Ans:0.4]

Quiz 2 Sep 24

ET Sep 24

Quiz 1 Jan 25

1. Consider the function $f(x, y) = \sin(xy)$. Let $L(x, y)$ be the linear approximation of $f(x, y)$ around the point $\left(\frac{\pi}{2}, 0\right)$. What is the value of $L\left(\frac{\pi}{2} + 0.1, 0.1\right)$? (Use $\pi = 3.14$)

   (a) 0.162

   (b) 0.257

   (c) 0.157

   (d) 1.57

   [Ans: c]

2. Which of the following statements is/are correct?

   (a) $f(x) = \ln(x)$ is continuous and differentiable in $[0, \infty)$.

   (b) $f(x) = e^x$ is continuous and differentiable for all $x \in \mathbb{R}$.

   (c) $f(x) = \sqrt{x}$ is continuous but not differentiable at $x = 0$.

   (d) $f(x) = \frac{1}{x}$ is continuous and differentiable for all $x \neq 0$.

   [Ans: b, c, d]

Quiz 2 Jan 25

ET Jan 25

Quiz 1 May 25

1. The function
$$f(x) = \begin{cases} px^2 + qx + 2, & x < 1 \\ rx + s, & x \geq 1 \end{cases}$$
is differentiable at $x = 1$. Which of the following is/are true?

   (a) $r = 2p + q$ and $s = 2 - p$

   (b) $p + q = r + s$

   (c) $q = r$ and $s = 2$

   (d) $p + q + 2 = r + s$ and $2p + q = r$

   [Ans: a, d]

2. Which of the following is/are true?

   (a) If $a_n = -1 + \frac{n}{n+1}$ then $a_n \to 0$

   (b) If $a_n = \sin(n)$ then $a_n \to 0$

   (c) If $a_n = n!$ then $a_n \to \infty$

   (d) $a_n = \sin(n)$ is not convergent

   [Ans: a, c, d]

3. Let $f(x, y) = \ln(x^2 + y^2)$. Use the linear approximation at the point $(1, 2)$ to estimate $f(1.02, 2.01)$. Write the answer correctly to 4 decimal places. [Ans: 1.625]

Quiz 2 May 25

ET May 25

Quiz 1 Sep 25

1. Consider $f(x) = e^{x_1} + x_2$.

   (a) Which is $L_{(0,0)}[f(x)]$?

   i. $1 + x_1 + x_2$
   ii. $x_1 + 2x_2$
   iii. $1 - x_1 - x_2$
   iv. $x_1 - x_2$

   [Ans: a]

   (b) Recall that $L_{(0,0)}[f](x)$ corresponds to a plane in $\mathbb{R}^3$ and can be represented as the set of all points that satisfy the equation $w^T x + b = 0$, where $w \in \mathbb{R}^3$, $b \in \mathbb{R}$, with w being normal to the plane. If $w^T w = 3$, what is b?

   [Ans: 1]

Quiz 2 Sep 25

ET Sep 25